

Basic L^AT_EX Template

William J. Turner

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Abstract

This paper computes the distance between two points and fits both linear and exponential functions through the two points.

1 Introduction

Consider the two points $(-1, 16)$ and $(3, 1)$. Section 2 computes the distance between these two points. Section 3 computes a linear equation $y = mx + b$ through the two points, and Section 4 fits an exponential equation $y = Ae^{kx}$ through the two points.

2 Distance

We can use the distance formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad (1)$$

to determine the distance between any two points (x_1, y_1) and (x_2, y_2) in \mathbb{R}^2 . For our example, $(x_1, y_1) = (-1, 16)$ and $(x_2, y_2) = (3, 1)$, so plugging these values into the distance formula (1) tell us the distance between the two points is

$$d = \sqrt{(3 - (-1))^2 + (1 - 16)^2} = \sqrt{4^2 + (-15)^2} = \sqrt{241}.$$

3 Linear Fit

Consider a linear equation $y = mx + b$ through the two points. We will first determine the slope m of the line in Section 3.1, and we will then determine the y -intercept b of the line in Section 3.2.

3.1 Slope

The slope of the line passing through the two points is given by the formula

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}.$$

Plugging in our two points, we find the slope of the line between them is

$$m = \frac{1 - 16}{3 - (-1)} = -\frac{15}{4}. \quad (2)$$

3.2 Intercept

To find the y -intercept of the line, we start with the point-slope form of the line of slope m through the point (x_0, y_0) :

$$y - y_0 = m(x - x_0).$$

We plug in the point $(x_0, y_0) = (-1, 16)$ and the slope we found previously (2) to obtain the equation

$$y - 16 = -\frac{15}{4}(x + 1).$$

Solving for y , we find the slope-intercept form of the line:

$$\begin{aligned} y &= -\frac{15}{4}x - \frac{15}{4} + 16 \\ &= -\frac{15}{4}x + \frac{49}{4}. \end{aligned}$$

Therefore, the y -intercept is $b = 49/4$, and the equation $y = -\frac{15}{4}x + \frac{49}{4}$ describes the line through the two points.

4 Exponential Fit

Let us consider the exponential function $y = Ae^{kx}$. For this function to pass through both points, we must find constants A and k that satisfy both equations $16 = Ae^{-k}$ and $1 = Ae^{3k}$. To solve these two simultaneous equations, we first take the ratio of the two equations, which gives us a single equation involving only k :

$$16 = \frac{Ae^{-k}}{Ae^{3k}} = e^{-4k}.$$

We can take the natural logarithm of this equation to solve for k :

$$-4k = \ln(16) = 4 \ln(2),$$

which means $k = -\ln(2)$.

We can then use this value of k , along with either of the two points to solve for A . Let us consider the point $(-1, 16)$:

$$16 = Ae^{(-\ln(2))(-1)} = Ae^{\ln 2} = 2A.$$

Solving for A , we find $A = 8$, and the exponential equation through both points is

$$y = 8e^{-\ln(2)x} = 82^{-x} = 8 \left(\frac{1}{2}\right)^x.$$